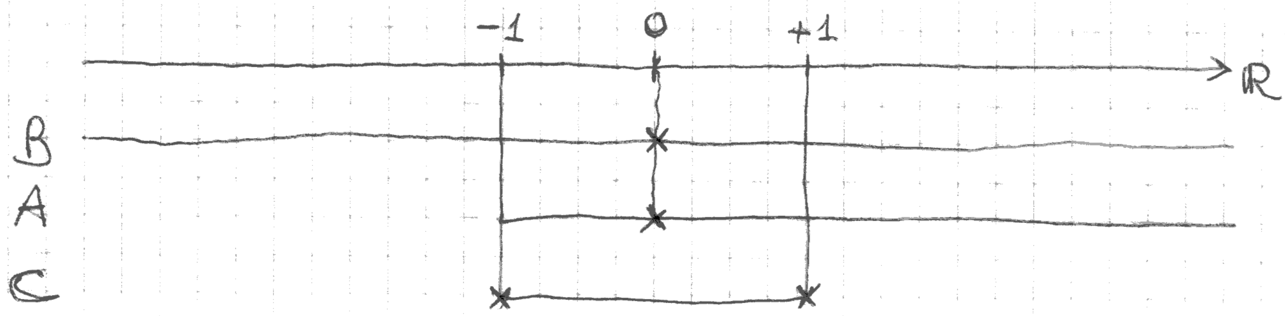
1) $A = [-1, +\infty) \setminus \{0\} = [-1, 0) \cup (0, +\infty)$

$B = CE(-\ln(x^2)) = \{x \in \mathbb{R} \text{ t.c. } x^2 > 0\} = \{x \in \mathbb{R} \text{ t.c. } x \neq 0\} = \mathbb{R} \setminus \{0\}$

$= (-\infty, 0) \cup (0, +\infty)$

$C = (-1, 1)$



a) $A \subseteq B$, sì.

$C \not\subseteq A$, perchè $0 \in C$ ma $0 \notin A$

b) $A \cup C = [-1, +\infty)$

$B \cup C = \mathbb{R}$

c) $A \cap B = A$, perchè $A \subseteq B$

$B \cap C = (-1, 0) \cup (0, +1)$

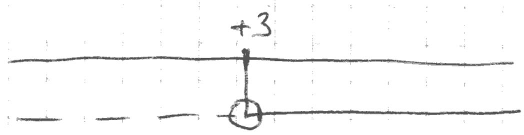
---

2) $\dfrac{1-x^2}{x-3} \leq \dfrac{3}{3-x} \iff \dfrac{1-x^2}{x-3} - \dfrac{3}{3-x} \leq 0$

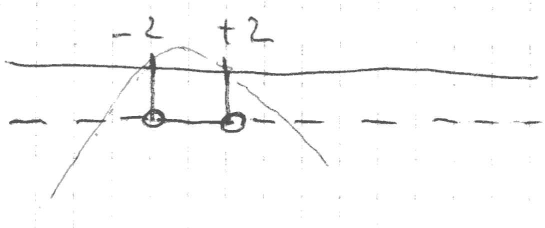
$\iff \dfrac{1-x^2}{x-3} + \dfrac{3}{x-3} \leq 0 \iff \dfrac{1+3-x^2}{x-3} \leq 0$

$\iff \dfrac{4-x^2}{x-3} \leq 0$

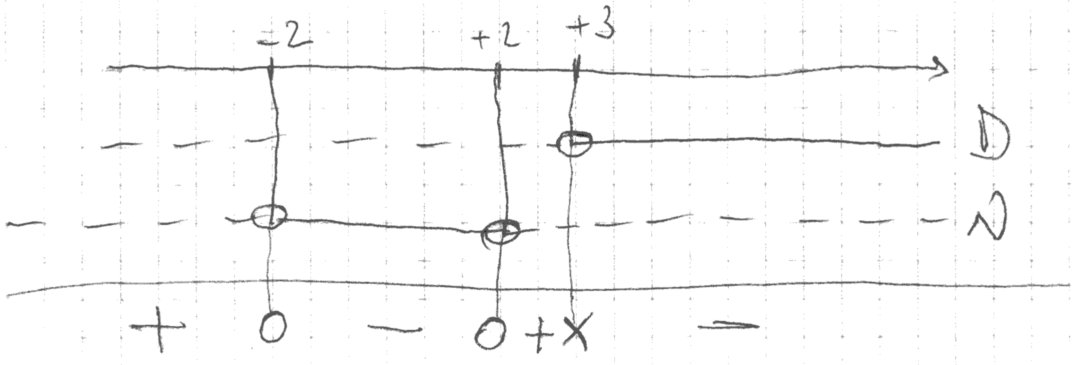
segno di $x-3$:



segno di $4-x^2$:



⇒ Segno di $\frac{4-x^2}{x-3}$:

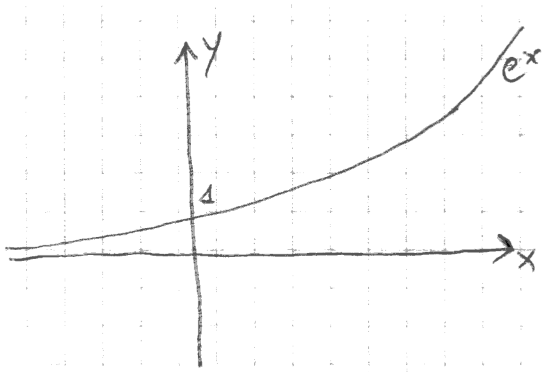


⇒ $S = [-2, +2] \cup (+3, +\infty)$

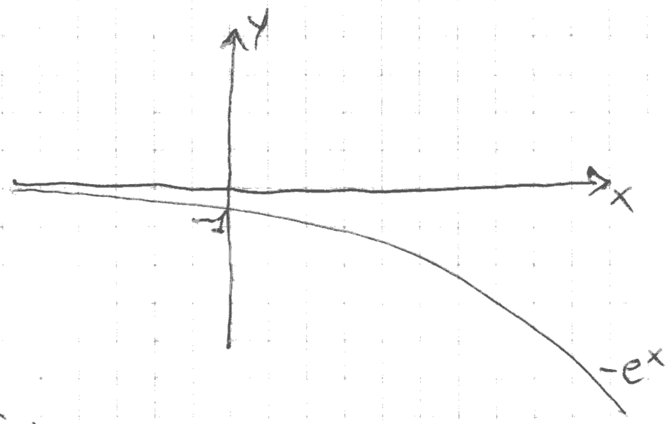
---

3) $f(x) = \begin{cases} \frac{1-x}{x} & \text{se } x<0 \\ -e^x & \text{se } x \geq 0 \end{cases}$

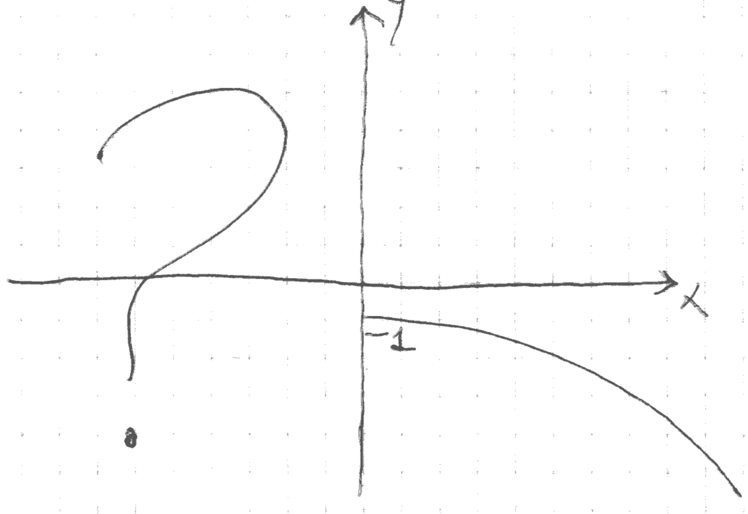
Il grafico di e^x è:



⇒ Il grafico di $-e^x$ è:



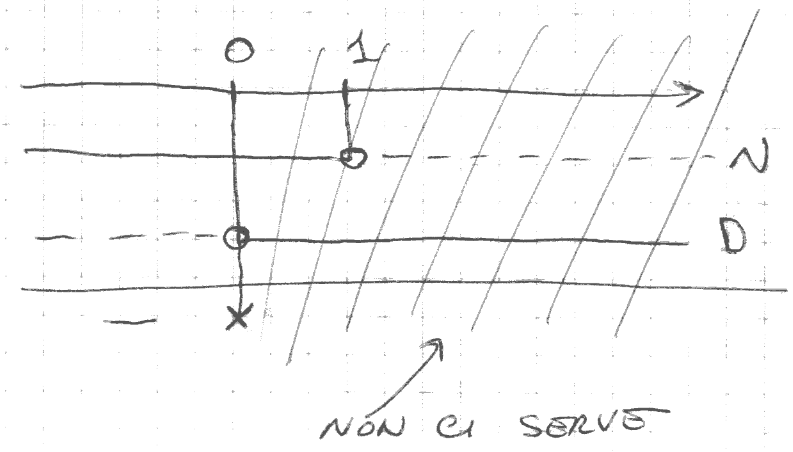
⇒ Il grafico di $f(x)$ è:



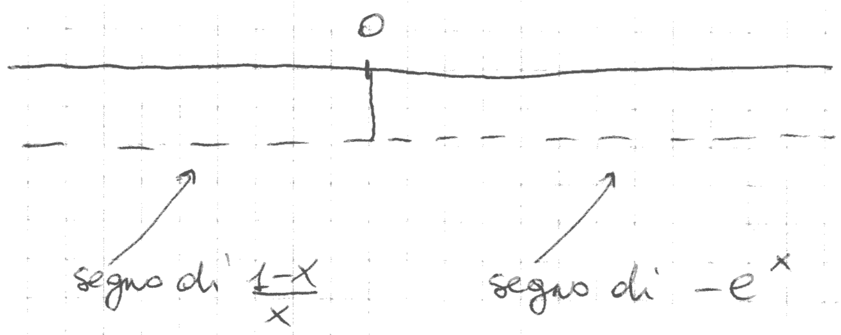
$\Rightarrow$ studio soltanto $\dfrac{1-x}{x}$ per $x<0$.

a) $CE\left(\dfrac{1-x}{x}\right) = \mathbb{R}\setminus\{0\} \Rightarrow CE(f) = (-\infty, 0) \cup [0, +\infty) = \mathbb{R}$

b) Segno di $\dfrac{1-x}{x}$: per $x<0$:



$\Rightarrow$ Segno di f:



c) $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{1-x}{x} = -1 \Rightarrow$ ASINTOTO ORIZZONTALE

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} -e^x =$ (NON SERVE FARLO, ABBIAMO GIÀ IL GRAFICO) $= -\infty$ DAL GRAFICO.

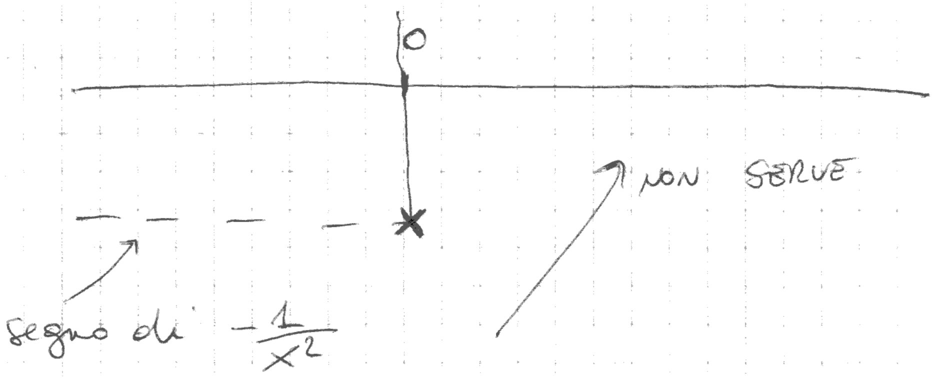
NON C'È ASINTOTO OBLIQUO, PERCHÈ SAPPIAMO COM'È FATTA e^x.

$\lim\limits_{x\to0^-} f(x) = \lim\limits_{x\to0^-} \dfrac{1-x}{x} = -\infty$ ASINTOTO VERTICALE

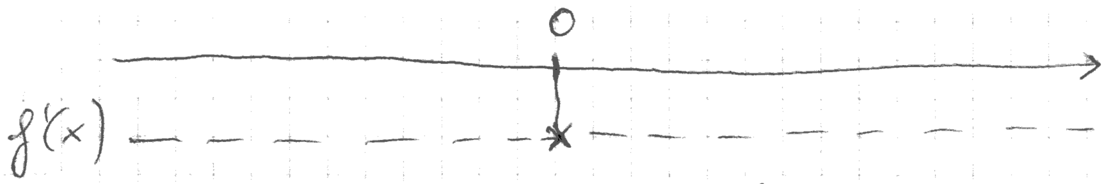
d) $f'(x) = \begin{cases} \cancel{\quad} \left(\frac{1-x}{x}\right)' = \left(\frac{1}{x}\right)' - (1)' = -\frac{1}{x^2} & x<0 \\ \text{non serve} & x>0 \end{cases}$

In 0: $\lim_{x\to 0^-} f'(x) = -\infty \Rightarrow \nexists\, f'(0)$

segno derivata:



$\Longrightarrow$ INTERVALLI DI CRESCENZA E DECRESCENZA:



$\Rightarrow$ f decresce in $(-\infty, 0)$ e $[0, +\infty)$

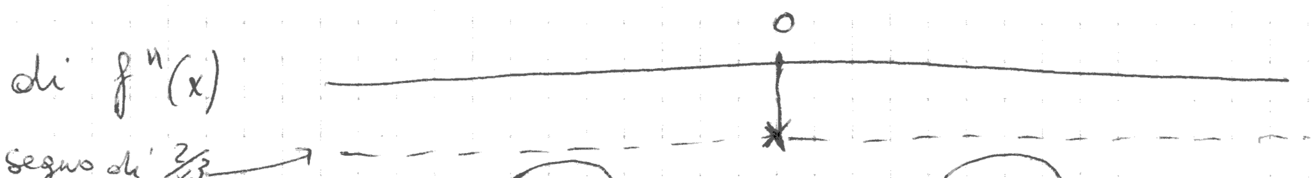
e) •) Poiché $\lim_{x\to 0^-} = -\infty$, NON C'È MINIMO GLOBALE

•) Il pto $(0,-1)$ è UN MAX LOCALE, perché A SX $f \to -\infty$ e a dx $f \searrow$

•) Il pto $(0,-1)$ è anche MAX GLOBALE, perché $\lim_{x\to +\infty} f(x) = -1$, e $f(x) \searrow$.

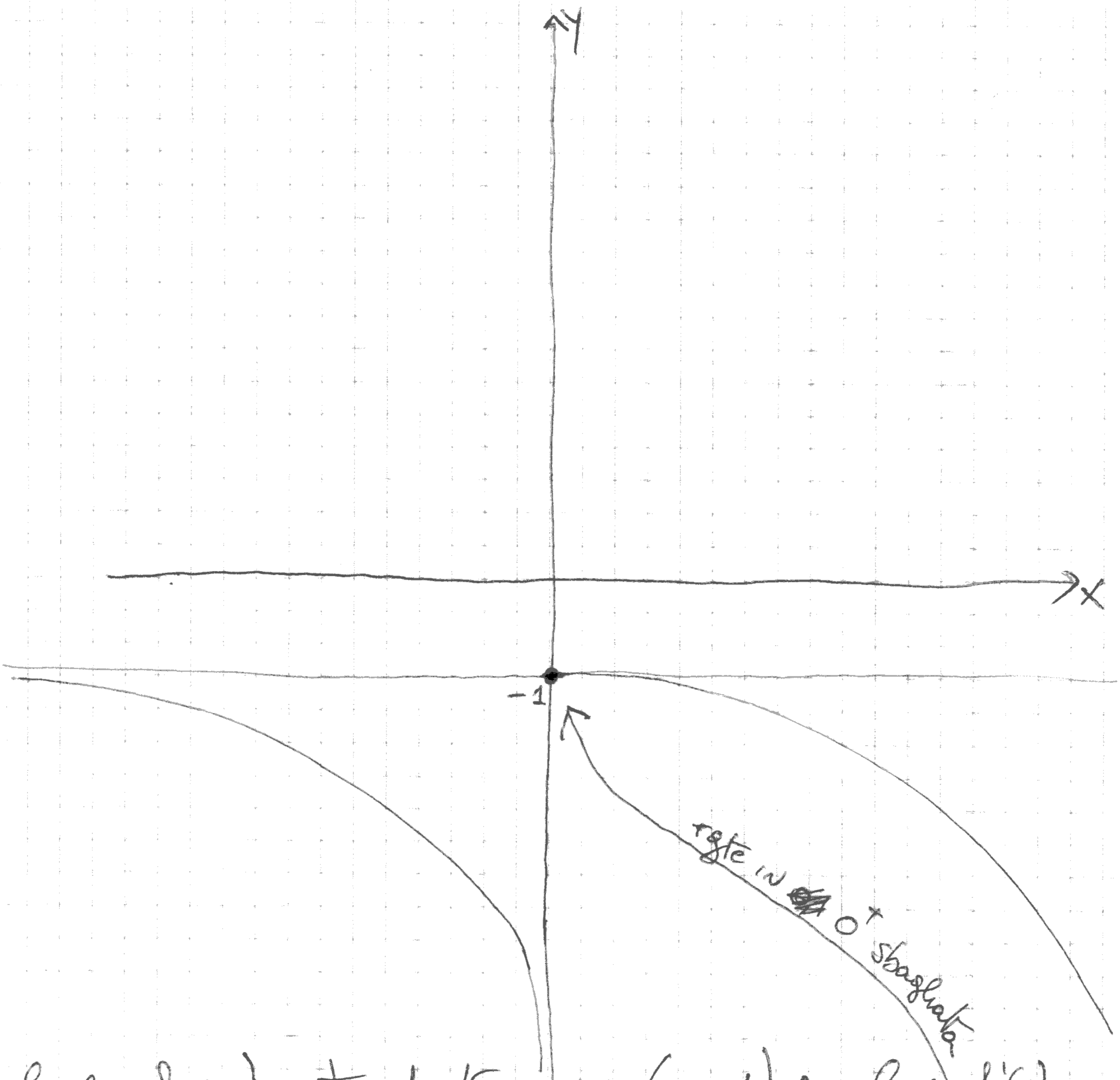
f) $f''(x) = \begin{cases} +2/x^3 & x<0 \\ \text{non serve} & x>0 \\ \nexists & x\neq 0 \ (\text{perché } \nexists\, f'(0)) \end{cases}$

segno di $f''(x)$



f, continua) $\Rightarrow$ non ci sono flessi, perchè $f''(x)$ non cambia mai di segno.

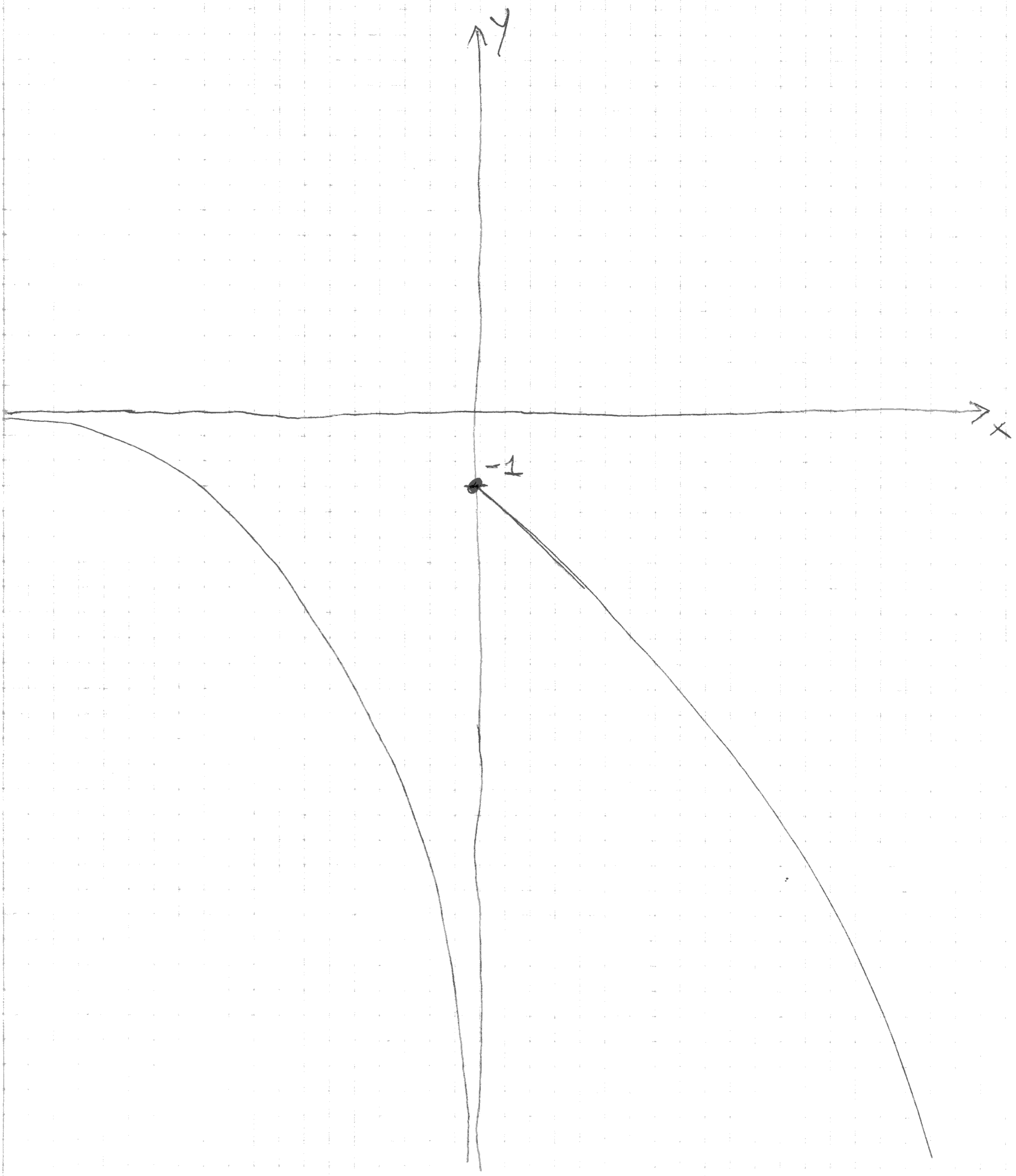
g)



Per il grafico): tg destra in $(0,-1)$: $\lim_{x \to 0^+} f'(x) =$

$= \lim_{x \to 0^+} (-e^x) = -1 \Rightarrow$ GRAFICO DA CORREGGERE

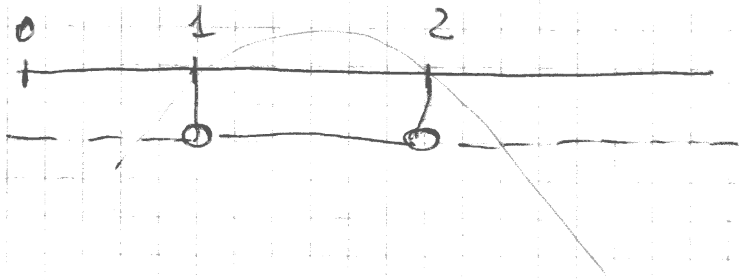
$\Rightarrow$ grafico di f:



4) Area $= \int |f|$, quindi studio il segno di $f(x)$ per $x \in (0,2)$.

$f(x) = -x^2+3x-2$, con $x_1 = 1$ e $x_2 = 2$

$\Longrightarrow$ segno di $f(x)$



$\Longrightarrow$ ~~Area =~~

$$\Longrightarrow |f(x)| = \begin{cases} -f(x) & \text{se } x \le 1 \\ f(x) & \text{se } 1 < x \le 2 \\ -f(x) & \text{se } x > 2 \end{cases}$$

$$\Longrightarrow A = \int_0^2 |f(x)|\, dx = \int_0^1 -f(x)\, dx + \int_1^2 f(x)\, dx =$$

$$= -\left[-\frac{x^3}{3}+\frac{3x^2}{2}-2x\right]_0^1 + \left[-\frac{x^3}{3}+\frac{3x^2}{2}-2x\right]_1^2 =$$

$$= -\left(-\frac{1}{3}+\frac{3}{2}-2\right) + \left(\left(-\frac{8}{3}+\frac{12}{2}-4\right)-\left(-\frac{1}{3}+\frac{3}{2}-2\right)\right) =$$

$$= -\left(\frac{-2+9-12}{6}\right) + \left(\frac{-8+18-12}{3} - \frac{-2+9-12}{6}\right) =$$

$$= \frac{5}{6} + \left(-\frac{2}{3}+\frac{5}{6}\right) = \frac{5}{6} + \frac{1}{6} = 1$$

<u>OSS</u> SE NON SI CAMBIA IL SEGNO di $\int_0^1$, l'Area viene: $-\frac{5}{6} + \left(-\frac{2}{3}+\frac{5}{6}\right) = -\frac{2}{3} < 0$

area NEGATIVA? IMPOSSIBILE

5) <u>OSS</u> $CE(f) = \{e^x - 1 \neq 0\} = \{e^x \neq 1\} = \{x \neq 0\}$

$\Rightarrow \int_0^1 f$ è UN INTEGRALE IMPROPRIO

TROVIAMO LA PRIMITIVA DI $\dfrac{e^x}{\sqrt{e^x-1}}$:

$$\int \frac{e^x}{\sqrt{e^x-1}}\,dx$$

$$= \int \frac{de^x}{\sqrt{e^x-1}} \overset{t=e^x}{=} \int \frac{dt}{\sqrt{t-1}} = \int (t-1)^{-\frac{1}{2}}\, d(t-1)$$

$$\overset{t-1=s}{=} \int s^{-\frac{1}{2}}\, ds = \frac{s^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{s^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{s}$$

$$\Rightarrow \int \frac{e^x}{\sqrt{e^x-1}}\,dx = 2\sqrt{e^x-1}$$

<u>VERIFICA</u>: $\left(2\sqrt{e^x-1}\right)' = 2\left(\dfrac{+1}{2\sqrt{e^x-1}}\right)\cdot e^x = \dfrac{e^x}{\sqrt{e^x-1}}$

ok

$$\Rightarrow \int_0^1 \frac{e^x}{\sqrt{e^x-1}}\,dx = \lim_{t \to 0^+} \left[2\sqrt{e^x-1}\right]_t^1 =$$

$$= 2\sqrt{e-1} - \lim_{t\to 0^+} 2\sqrt{e^t-1} = \boxed{2\sqrt{e-1}}$$

6) $\sum_{n=1}^{+\infty} \frac{5^{n+1}}{2^n} = 5 \sum_{n=1}^{+\infty} \left(\frac{5}{2}\right)^n \Rightarrow$ NON CONVERGE, perché $\lim_{n \to +\infty} \left(\frac{5}{2}\right)^n \neq 0$

$\sum_{n=2}^{+\infty} \frac{3}{n^2} = 3 \sum_{n=2}^{+\infty} \frac{1}{n^2} \Rightarrow$ converge, perché serie armonica generalizzata con $\alpha > 1$

---

7) a) $\det A = 0$ perché A ha due colonne proporzionali: $2 \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$

$\Rightarrow A$ non è invertibile.

b) $A \cdot B = \begin{pmatrix} 2 & -2 \\ 4 & -4 \\ 6 & -6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

c) Poiché $\det A = 0$, $rkA < 3$. Poiché $\det \begin{pmatrix} 2 & -13 \\ 4 & -18 \end{pmatrix} = -36 + 52 \neq 0$, $\boxed{rkA = 2}$

$rkB = 1$, perché $\det(2) \neq 0$